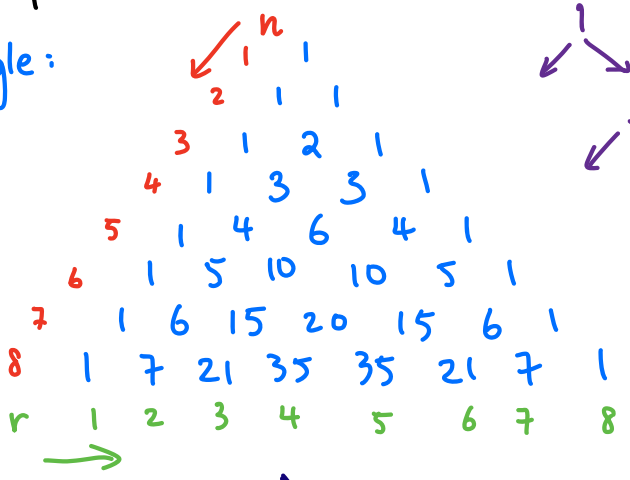
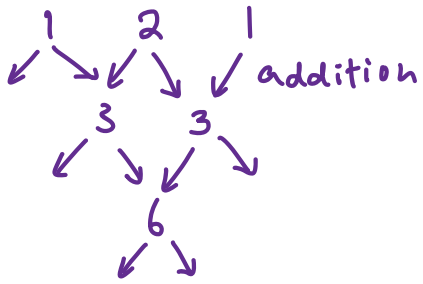
Binomial Expansion

Pascal's triangle:





$${}^nC_r \text{ or } \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$$ ↑ use the triangle to find it (or calculator)

Binomial expansion:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^r b^{n-r} = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$$

$$= a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \ldots + \binom{n}{n-2} a^2 b^{n-2} + \binom{n}{n-1} a b^{n-1} + b^n$$

Example: $(a+b)^2 = a^2 + \binom{2}{1} ab + b^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + \binom{3}{1} a^2 b + \binom{3}{2} ab^2 + b^3 = a^3 + 3a^2 b + 3ab^2 + b^2$

More on Combinations & Permutations

Combinations

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$ = # of ways r items can be chosen from n items

"n choose r"

☆ Order is NOT important

e.g. Apple (A) Banana (B) Orange (O) (list of n=3)

What is the # of combinations of 2 items? → AB, BO, AO (3 combinations) ${}^3C_2 = 3$

Permutations

$${}^nP_r = \underbrace{n(n-1)\ldots(n-r+1)}_{r \text{ factors}} = \frac{n!}{(n-r)!}$$ = # of ways r items can be permuted from n items

☆ Order IS important

e.g. A, B, O (list of n=3)

# of permutations of 2 items → AB, BA, BO, OB, AO, OA (6 combinations) ${}^3P_2 = 6$

Challenge (p.164)

a) $(a+b)^4 - (a-b)^4 = \left[(a+b)^2 + (a-b)^2\right]\left[(a+b)^2 - (a-b)^2\right]$

$= \left[a^2+2ab+b^2 + a^2-2ab+b^2\right]\left[a^2+2ab+b^2 - a^2+2ab-b^2\right]$

$= \left[2a^2+2b^2\right]\left[4ab\right] = 8ab(a^2+b^2)$

b) $\begin{matrix} a+b=17 \\ a-b=5 \end{matrix} \Rightarrow \begin{matrix} a=11 \\ b=6 \end{matrix}$ $\quad 82896 = (11+6)^4 - (11-6)^4 = 8(11)(6)(11^2+6^2)$

$= (11)^3(6)(8) + (11)(6)^3(8)$

Estimating using binomial expansion

$\Rightarrow \left(1+\frac{x}{4}\right)^8 = 1+2x+\frac{7}{4}x^2+\frac{7}{8}x^3+\ldots$

what is 1.025^8?

$1+\frac{x}{4} = 1.025$

$x = 0.1$

$1+0.2+\frac{7}{40}+\frac{7}{80}+\ldots$

≈ 1.2184

$\Rightarrow \left(1+\frac{x}{2}\right)^{10} = 1+5x+\frac{45}{4}x^2+15x^3+\ldots$

what is 1.005^{10}?

$1+\frac{x}{2} = 1.005$

$x = 0.01$

$1+0.05+\frac{45}{4}(0.01)^2+15(0.01)^3+\ldots$

≈ 1.05114